Heterogeneity and the Price Level

Lecture based on *Price Level and Inflation Dynamics in Heterogeneous Agent Economies* by Kaplan, Nikolakoudis, and Violante (2026)

Manuel Macera | 2026

Price level determination

- In models where nominal government debt is the only asset, the price level depends on monetary and fiscal policy jointly.
- With an interest-rate rule and surpluses that do not respond to debt, the government budget constraint pins down the price level.
- The constraint, in nominal and real terms:

$$B_{t+1} = (1 + i_t)\left[B_t - s_t P_t Y\right] \quad \Longrightarrow \quad \boxed{b_{t+1} = (1 + r_t)\left(b_t - s_t\right)}$$

  where $b_t = B_t/(P_t Y)$ is real debt, s_t the real surplus and r_t the real rate.
- Iterate forward, and rule out debt that grows faster than r:

$$\boxed{\frac{B_0}{P_0 Y} = \sum_{k \geq 0} \frac{s_k}{\prod_{m<k}(1 + r_m)}\,.}$$

- Inherited debt, in real terms, equals the present value of its surpluses.

The FTPL with a representative agent

- Take a constant surplus s, a constant real rate $r > 0$ and inherited nominal debt B_0. The valuation equation gives

$$P_0 = \frac{B_0}{Y} \frac{r}{(1+r)\, s}.$$

- This is the **fiscal theory of the price level** (FTPL, Woodford (1995) and Sims (1994)). The path of prices is recovered using the Fisher equation:

$$1 + \pi_{t+1} = \frac{1+i_t}{1+r_t}, \qquad P_t = P_0 \prod_{k<t} (1+\pi_{k+1}).$$

- With a representative agent, the Euler equation and market clearing fix the real rate at the discount rate: $r_t = \rho$.

Deficits in the representative agent FTPL

- Except in 1998–2001, the US has run a primary deficit every year since 1970.
- With a constant deficit $s < 0$ and $r = \rho > 0$, the present value of surpluses is $s(1+\rho)/\rho < 0$.
- A positive real value of debt cannot equal a negative present value: **no positive price level exists**.
- Yet governments sustain deficits with a stable debt ratio. When is that possible?

Sustainable deficits

- Constant debt, $b_{t+1} = b_t$ in the budget constraint, requires

$$s = \frac{r}{1+r}\, b\,.$$

- $r > 0$: interest adds to the debt, and constant debt needs **surpluses**.
- $r < 0$: the debt erodes by itself, so constant debt allows **deficits**.
- A representative agent rules this out, since $r = \rho > 0$. Who would hold debt with a negative real return?

Precautionary demand for debt

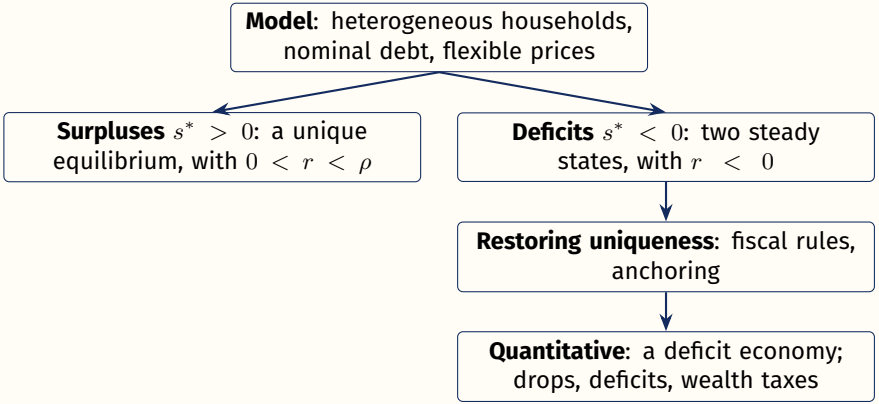
- With uninsurable income risk, households want a safe asset to self-insure.
- When government debt is the only alternative, they hold it even at a negative return: their demand $a(r)$ is **positive for some** $r < 0$.
- A steady state with deficits can then exist. Fiscal policy sets the sign of r: $0 < r < \rho$ with surpluses, $r < 0$ with deficits.
- But with $r < 0$ the sum $\sum_k s/(1+r)^k$ does not converge: the valuation equation **no longer pins** P_0. What does?

This paper

- The paper builds an economy with three features:
  - a fiscal authority issues **nominal debt** to finance transfers;
  - a monetary authority sets the **nominal rate** on that debt;
  - households face **uninsurable income risk** and save for precaution.
- The first two are the FTPL's setting; the third is the Bewley–Huggett–Aiyagari tradition.

- Prices are flexible and output is an endowment, so the focus is on how the price level is determined.

Road map



- A question to carry through the lecture: with the same total debt and the same surpluses, does it matter for the price level **who holds the debt**?

The environment

The paper is in continuous time; this lecture is in discrete time

Object	Paper (continuous time)	Lecture (discrete time)
Income shocks	Poisson intensities $\lambda_{z,z'}$	Markov matrix $\Pi(z, z')$
Impatience	discount rate ρ	discount factor $\beta = 1/(1+\rho)$
Household problem	HJB equation	Bellman equation
Distribution	Kolmogorov forward equation	$\mu_{t+1} = \mathcal{T}_t \, \mu_t$
Debt	$\dot{b}_t = r_t b_t - s_t$	$b_{t+1} = (1 + r_t)(b_t - s_t)$
Saddle path	one positive eigenvalue	one eigenvalue above 1

- The mechanisms carry over; nothing in the argument needs continuous time

Households: endowment risk and a nominal bond

- Output $Y_t = y_0(1+g)^t$; household j receives the share z_{jt}, with $\int z_{jt}\, dj = 1$
- z follows a Markov chain $\Pi(z, z')$ with N states
- The only asset: a one-period nominal bond. With A_t the nominal balance at the start of t,

$$A_{t+1} = (1+i_t)\big[A_t + P_t Y_t\,(z_t - \tau_t(z_t)) - P_t \tilde{c}_t\big], \qquad A_{t+1} \geq 0$$

- $\tau_t(z)$: taxes net of transfers; the government controls their level *and* distribution
- Preferences: $\mathbb{E}_0 \sum_t \tilde{\beta}^t\, \tilde{c}_t^{1-\gamma}/(1-\gamma)$

In detrended real terms, one rate matters

- Divide by $P_t Y_t$: $a_t = A_t/(P_t Y_t)$, $c_t = \tilde{c}_t / Y_t$. Then

$$a_{t+1} = (1 + r_t)\big[a_t + z_t - \tau_t(z_t) - c_t\big], \qquad 1 + r_t = \frac{1 + i_t}{(1 + \pi_{t+1})(1 + g)}\,.$$

- Bellman equation, with the growth-adjusted discount factor $\beta = \tilde{\beta}(1+g)^{1-\gamma}$:

$$V_t(a, z) = \max_{c,\, a'} \frac{c^{1-\gamma}}{1-\gamma} + \beta \sum_{z'} \Pi(z, z')\, V_{t+1}(a', z')$$
$$\text{s.t. } a' = (1 + r_t)\big[a + z - \tau_t(z) - c\big], \quad a' \geq 0\,.$$

- Euler equation, with equality when $a' > 0$: $c_t^{-\gamma} \geq \beta(1 + r_t)\, \mathbb{E}_t\, c_{t+1}^{-\gamma}$.
- Initial real wealth, $a_{j0} = A_{j0}/(P_0 Y_0)$, depends on P_0.

Government: nominal debt, real surpluses, a rate peg

- Nominal debt $B_0 > 0$ is given, and $b_t \geq 0$: the government borrows, never lends. With surplus $s_t = \int \tau_t(z_{jt})\, dj$,

$$b_{t+1} = (1 + r_t)\,(b_t - s_t)\,.$$

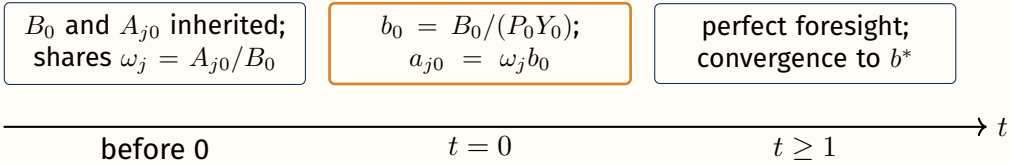
- Baseline: a fixed tax function $\tau^*(z)$, so $s_t = s^*$, and a rate peg $i_t = i^*$.

- Fiscal and monetary policy do not change over time, so any movement in r_t or b_t must come from the **initial state**.

- With the peg, the Fisher equation turns every real rate into an inflation rate:

$$\pi_{t+1} \approx i^* - r_t - g\,.$$

Initial conditions



- B_0 is **predetermined**; P_0 is not. The equilibrium value of real debt b_0 pins down the price level.
- From any initial distribution of shares, with perfect foresight, the economy converges to the stationary equilibrium of the fiscal and monetary policy in place.
- So equilibrium can be defined in real terms, with b_0 as an unknown.

Equilibrium

Real equilibrium

Given the tax function $\tau^*(z)$ and the initial distribution of shares $f_0(\omega, z)$, a real equilibrium consists of real rates $\{r_t\}$, real debt $\{b_t\}$, household policy functions and distributions $\{f_t\}$ such that

- households optimize, given $\{r_t\}$ and initial wealth $a_{j0} = \omega_j b_0$;
- f_t evolves with their choices;
- $b_{t+1} = (1 + r_t)(b_t - s^*)$ and $b_t \geq 0$;
- the goods market clears at every $t \geq 0$: $C_t = \int c_{jt}\, dj = 1$.

- Budgets imply $a_{t+1} - b_{t+1} = (1 + r_t)(1 - C_t)$; with $a_0 = b_0$, the bond market clears too.
- Each real equilibrium gives $P_0 = B_0/(b_0 Y_0)$ and $\pi_{t+1} \approx i^* - r_t - g$: a unique real equilibrium means a unique price path.

Computing an equilibrium

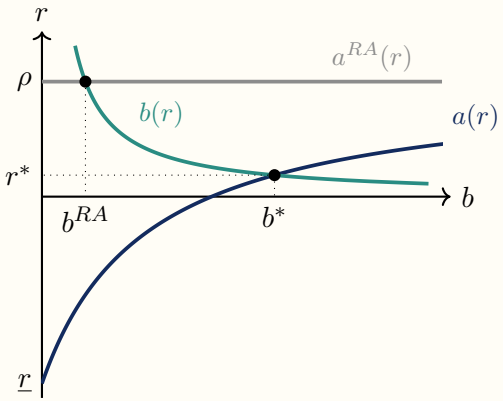
1. Solve the stationary equilibrium of the policy in place: r^* and b^*.
2. Guess a path $\{r_t\}$, with $r_t = r^*$ from some date T on.
3. Supply: the economy must reach $b_T = b^*$, so the valuation equation gives

$$b_0 = \sum_{k<T} \frac{s^*}{\prod_{m<k}(1+r_m)} + \frac{b^*}{\prod_{m<T}(1+r_m)},$$

   and the budget constraint gives $\{b_t\}$.
4. Demand: solve the household problem backward given $\{r_t\}$; start the distribution at $a_{j0} = \omega_j b_0$ and push it forward to get $\{C_t\}$.
5. Where $C_t < 1$, demand is too low and households save too much: lower r_t. Where $C_t > 1$, raise it. Repeat until $C_t = 1$ at every t, including $t = 0$.

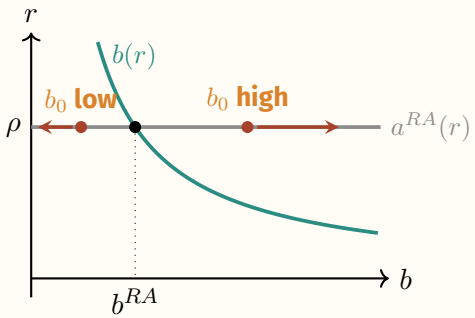
Primary surpluses: $s^* > 0$

Steady state with surpluses



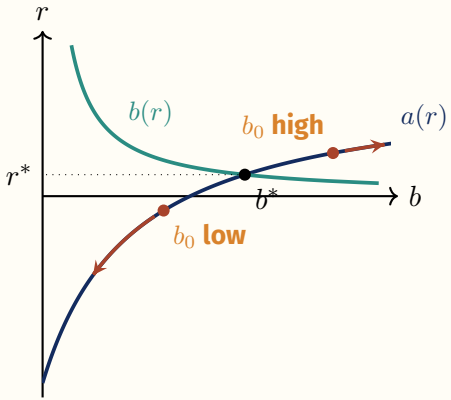
- Supply: a constant debt ratio requires $b(r) = s^*(1+r)/r$, a decreasing function of r.
- RA: demand is flat at $r = \rho$, and the equilibrium debt ratio is $b^{RA} = b(\rho)$.
- HA: $a(r)$ increases, explodes as $r \to \rho$, and is zero below some $\underline{r} < 0$.
- Steady state: $r^* < \rho$ and $b^* > b^{RA}$.

Uniqueness in the RA economy



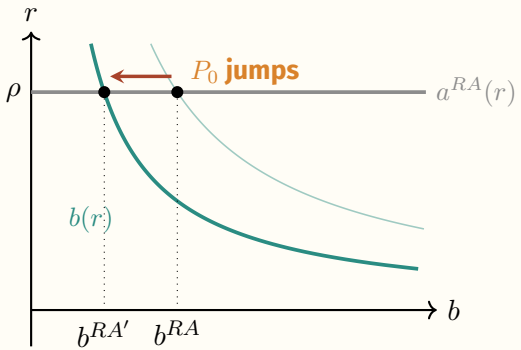
- With $r = \rho$, the steady state is $b^{RA} = b(\rho)$. Is any other b_0 an equilibrium?
- $b_0 < b^{RA}$: surpluses repay the debt, and then $b_t \geq 0$ **fails**.
- $b_0 > b^{RA}$: debt explodes, and households hold wealth they never consume: **not optimal**.
- Only $b_0 = b^{RA}$ survives: $P_0 = B_0/(b^{RA}Y_0)$.

Uniqueness in the HA economy



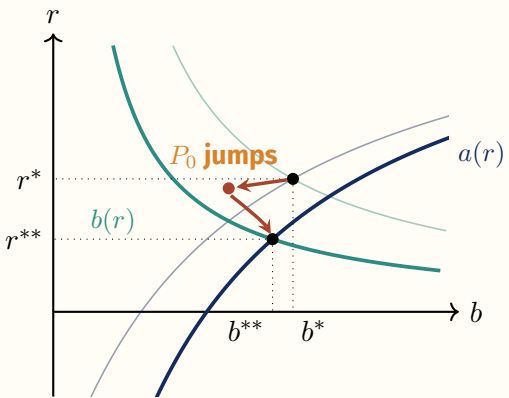
- Households hold debt only at the rate on their demand curve.
- $b_0 > b^*$ needs $r_0 > r^*$ for households to hold the bonds. At r_0, surpluses do not cover interest and debt grows, so r must rise further: debt explodes, **not optimal**.
- $b_0 < b^*$ needs $r_0 < r^*$. At r_0, surpluses exceed interest and debt shrinks, so r falls further: debt reaches zero, $b_t \geq 0$ **fails**.
- Only $b_0 = b^*$ survives: $P_0 = B_0/(b^* Y_0)$.

Lower surpluses in the RA economy



- Start at b^{RA}; surpluses fall permanently to $\Delta_s s^*$, with $\Delta_s < 1$.
- Supply shifts left: the new steady state $b^{RA'}$ has less real debt at the same $r = \rho$.
- P_0 jumps once, by the factor $1/\Delta_s$, and real debt falls to $b^{RA'}$ at once.
- r stays at ρ, so with the peg **inflation does not change**.

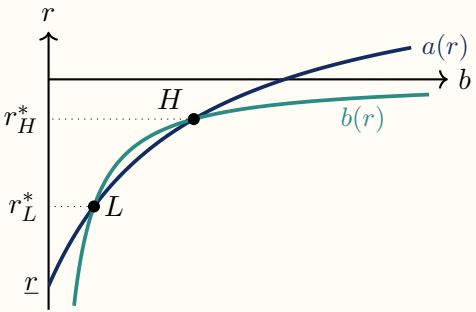
Lower surpluses in the HA economy



- Supply shifts left. Lower proportional taxes make after-tax income more volatile, so precautionary saving rises: demand shifts right.
- Each point on a demand curve has its own distribution of shares. The old shares adjust slowly: only b_0 moves at once.
- So r falls gradually to r^{**} as households trade and the distribution adjusts.
- With the peg, inflation **rises gradually**, ending $r^* - r^{**}$ higher.

Primary deficits: $s^* < 0$

Steady states with deficits

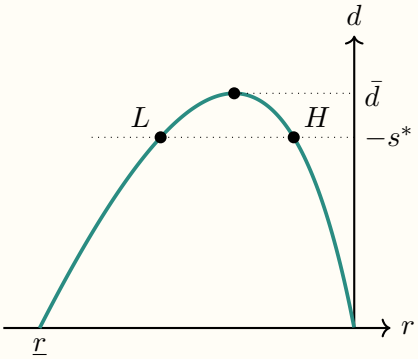


- Supply: constant debt with $s^* < 0$ needs $r < 0$, so $b(r) = s^*(1 + r)/r$ is now an increasing function of r.
- Demand has the same shape and reaches zero at $\underline{r}$.
- Generically **two** steady states, H and L, or none.
- H has more debt and a higher r, so lower inflation; L the reverse.

Households pay the government to hold its debt

- With $r < 0$, households make real interest payments that finance the deficit.
- They accept a negative return because government debt is their only way to self-insure.
- The steady states are **Pareto ranked**: H dominates L for every household.
  - aggregate consumption is the same in both (endowment economy);
  - with more real assets, households smooth consumption better.

A Laffer curve for deficits

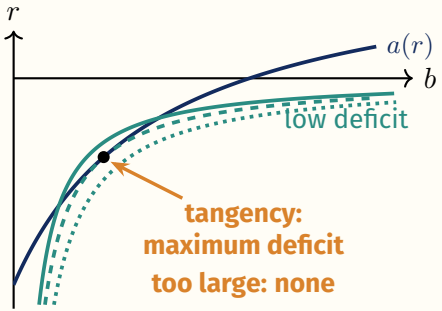


- Deficits are financed by negative real rates: a tax $-r/(1+r)$ on the base $a(r)$,

$$d(r) = -\frac{r}{1+r}\, a(r)\,.$$

- The same deficit can be financed in two ways: much debt and a mildly negative rate (H), or little debt and a very negative rate (L).
- A lower r raises the tax but shrinks the base, so $d(r)$ has a **maximum** $\bar{d}$.

Maximum sustainable deficit

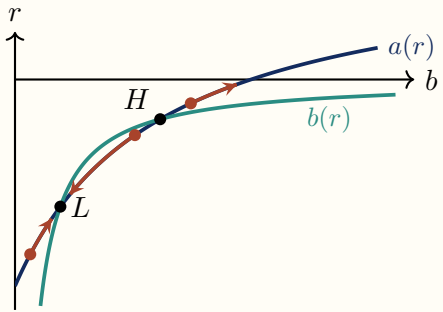


- Larger deficits shift $b(r)$ down and to the right.
- The maximum is at the tangency, where $d'(r) = 0$:

$$\frac{a'(r)\,r}{a(r)} = -\frac{1}{1+r}\,.$$

- A larger deficit has **no steady state**.
- Less uninsured risk shrinks the base $a(r)$, so the maximum deficit is **lower**. With no risk, $a(r) = 0$ for $r < \rho$: no deficit can be financed.

Non-stationary equilibria with deficits



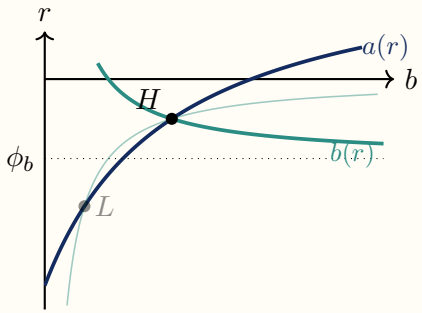
- Besides the two steady states, there is a **continuum of equilibria** converging to L.
- Above H, a higher r collects less than the deficit: debt grows and r rises further. **No equilibrium.**
- Below H, every b_0 converges to L: between H and L, a lower r collects more and debt shrinks; below L, the small base collects too little and debt grows.
- Only $b_0 = b_H$ leads to H. Every $b_0 < b_H$ gives a different P_0: which one does the economy select?

Equilibrium selection

Route	What it does
Real debt rule, $s_t = s^* + \phi_b(b_t - b_H)$	cuts deficits when debt falls: removes L
Real rate rule, $s_t = s^* + \phi_r(r_t - r_H)$	cuts deficits when r falls: removes L
Long-run inflation anchoring	beliefs rule out every path to L

- The rules change the steady states, so only H is left. Anchoring keeps both and rules out every path to L.

Real debt rule

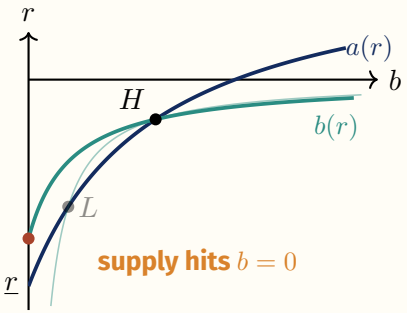


- Deficits respond to debt: $s(b) = s^* + \phi_b\,(b - b_H)$.
- Constant debt is the present value of its surplus, $b = \frac{1+r}{r}\,s(b)$:

$$\frac{r}{1+r} = \frac{s(b)}{b}\,.$$

- Supply **slopes down** if $\phi_b < r_H/(1+r_H)$.
- Then the curves cross only at H: low debt triggers smaller deficits, and debt keeps falling.

Real rate rule



- Deficits respond to the rate: $s(r) = s^* + \phi_r (r - r_H)$.
- Constant debt is the present value of its surplus:

$$b(r) = \frac{1+r}{r} s(r) .$$

- Supply reaches zero before demand does, so the curves cross only at H, if $\phi_r \leq s^*/(r_H - \underline{r})$.
- A falling r triggers smaller deficits, which speeds up the fall in debt: the path to L is gone.

Long-run inflation anchoring

- The central bank has two pillars: a path for i_t, and a long-run target π^* that only coordinates beliefs.

- The path to H is the unique equilibrium if
  - $i^* - \pi^* - g = r^*_H$: the target is consistent with H;
  - fiscal policy keeps H a saddle (a constant deficit does);
  - private beliefs about long-run inflation equal π^*.
- The authors call the third condition **"a big 'if' "**: nothing in the model makes beliefs settle there.

- P_0 and π_t stay endogenous, and fiscal policy still has to make H a saddle.

Quantitative experiments

Quantitative model

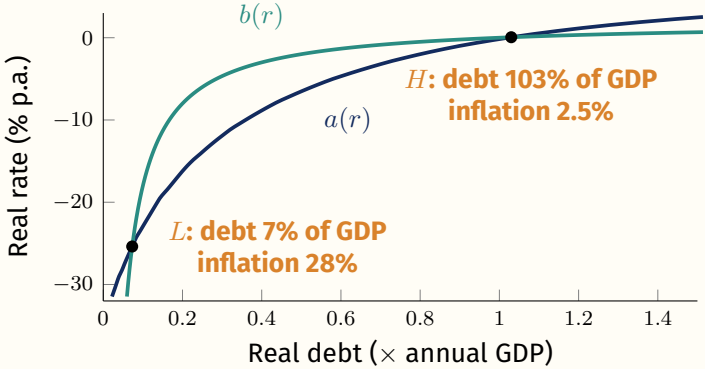
- **Unsecured credit**: households may borrow up to a real limit, paying a wedge over the saving rate. A higher price level now helps debtors.
- **Long-term debt**: a constant maturity rate. It matters when the nominal rate moves.
- Taxes and transfers, $\tau(z) = -\tau_0 + \tau_1 z$: a lump-sum transfer and a proportional tax. Government consumption x is a fixed share of output.
- Targets: US averages for 2014–2019, before the pandemic.
- From here on, numbers come from the paper's continuous-time model.

Calibration

Parameter	Value	Target
γ, inverse EIS	1	
ρ, discount rate	5.3% p.a.	debt / annual GDP $= 1.03$
g, growth	2.0% p.a.	postwar average
s.d. of log quarterly earnings	1.2	US micro data
borrowing limit; wedge	$-$\$18,600; 16% p.a.	credit card limits and rates
τ_1; τ_0	32%; 20% of GDP	tax share; $s^* = -2.0\%$ of GDP
x, government consumption	14% of GDP	US share
debt maturity rate	20% p.a.	5-year duration
i^*, nominal rate	2.6% p.a.	inflation of 2.5%

- Implied: $r = s^*/b^* \approx -1.9\%$, so the real rate is $r + g \approx 0.1\%$ and inflation $i^* - (r + g) = 2.5\%$.

Calibrated steady states



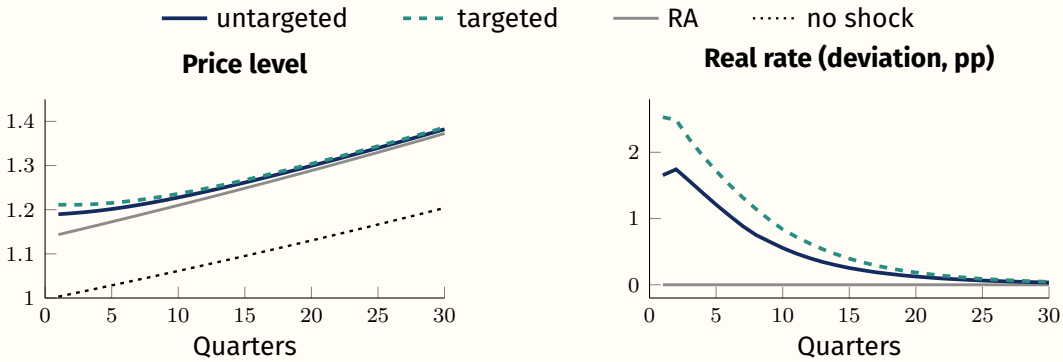
The experiments start at H and follow the saddle path, which assumes anchored long-run inflation.

Fiscal space

Deficit expanded through	$a(r)$	Max. deficit
baseline, % of GDP		2.0
government consumption	unchanged	4.34
lump-sum transfers	left: more insurance	4.17
lower proportional tax	right: more risk	4.61
transfers, no proportional tax or spending		9.30
transfers, no household borrowing		5.80

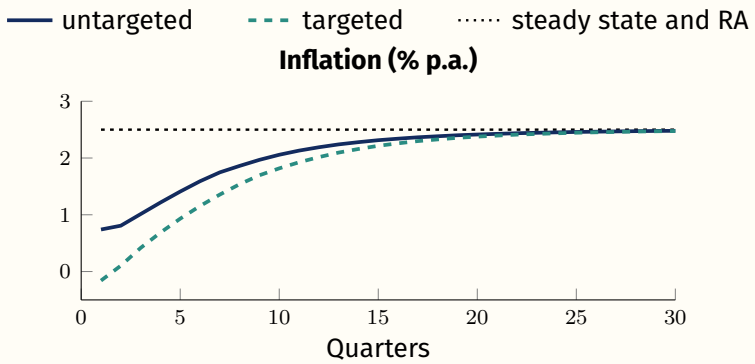
- The cap depends on **how** deficits grow: each instrument shifts precautionary demand, the base of the Laffer curve.
- Looser credit, more progressive taxes and more public insurance **reduce** the deficit the government can sustain.

Helicopter drop



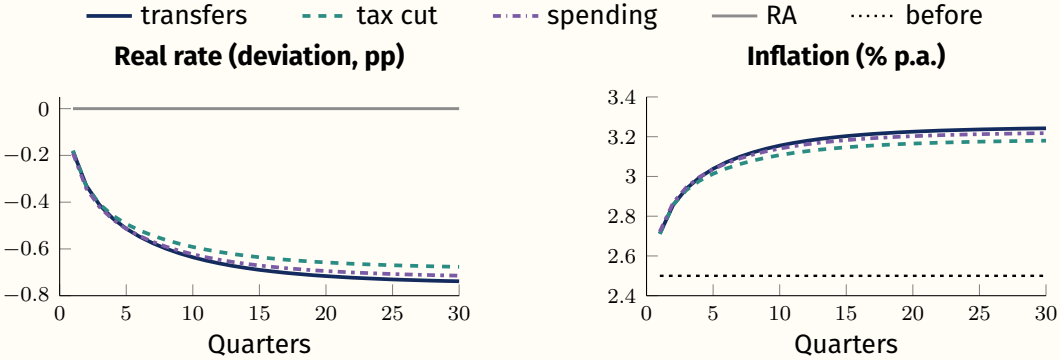
- New nominal debt of 14%, paid lump sum: to everyone (untargeted) or to the bottom 60% (targeted).
- High-MPC households sell the new bonds to consume: r rises until others hold them, higher interest needs less real debt, and P_0 jumps **more**: about 19% untargeted and 22% targeted, against 14% with RA.

Helicopter drop: inflation



- With the peg, the higher real rate **lowers** inflation after the jump.
- The cumulative rise in prices ends about the same as with RA: the difference is timing.

Permanent deficit expansion



- The deficit rises from 2.0% to 2.5% of GDP, starting at H. With RA, r does not move.
- In HA, r falls for good, so with the peg inflation stays **higher for good**. Holding it at 2.5% requires cutting i^* by the fall in r.

Limitations

Limitations the authors flag

- **Flexible prices**: the price level jumps at $t = 0$; in reality the burst is spread over time.
- **Endowment economy**: with labor supply, output falls after a deficit expansion and the price jump grows.
- **One asset**: no high-return productive asset, as in Kaplan, Moll, and Violante (2018).

Discussion

Discussion

Does a more progressive tax system raise or lower inflation?

- By the paper's comparative statics at H, with the peg fixed: it **raises** inflation.
- More insurance shifts $a(r)$ left; at H, the real rate falls.
- By $\pi \approx i^* - r - g$, a lower r means higher steady-state inflation.
- It also lowers the maximum sustainable deficit.
- The paper shows this for transfers, tax cuts and government consumption; a full progressive reform is beyond its experiments.

Discussion

Which steady state should we expect?

- Welfare points to H: it Pareto dominates L.
- So does continuity: moving surpluses gradually into deficits leads to H.
- Stability points the other way: L is locally stable and reached from a continuum of P_0.
- The paper selects H by assuming anchored long-run inflation, and calls that assumption **"a big 'if' "**.

Takeaways

1. With incomplete markets, valued government debt and persistent deficits can coexist: the growth-adjusted rate is negative ($r < g$).
2. Deficits produce two Pareto-ranked steady states; pinning down the price level needs fiscal rules or anchored beliefs.
3. There is a **maximum deficit**, and it shrinks with more social insurance.
4. **Redistribution** moves the price level: a helicopter drop raises P_0 more than with RA, through high MPCs and a higher r.

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